

# PATH LIFTING METHODS AND SOLUTIONS OF UNIVARIATE POLYNOMIAL EQUATIONS

L. J. HERNÁNDEZ PARICIO, M. MARAÑÓN GRANDES AND M. T. RIVAS RODRÍGUEZ

ABSTRACT. Given an analytic function  $h: Y \rightarrow B$  between Riemann surfaces (1-dimensional connected complex manifolds) with a known finite set of critical points  $C(h)$  and its corresponding set of critical values  $V(h) = h(C(h))$ , we can consider the covering map  $h: Y \setminus h^{-1}(V(h)) \rightarrow B \setminus V(h)$  whose Fox's completion is the branched covering map  $h$ . As a consequence of Fox's branched covering theory one has the lifting property for paths  $\beta: [0, 1] \rightarrow B$  such that  $\beta^{-1}(V(h)) \subset \{1\}$  and a transitive action of the fundamental group of  $B \setminus V(h)$  on  $h^{-1}(b_0)$  for  $b_0 \in B \setminus V(h)$ .

In this talk, we analyse the particular case of a complex map  $h: \mathbb{C} \rightarrow \mathbb{C}$  given by a polynomial of degree  $n \geq 1$ . We use piecewise linear paths (discrete paths) and lifting paths given by numerical Newton methods for simple and multiple roots to design an algorithm that gives all the solutions of the equation  $h(x) = b_0$  and, if  $b_0$  is not a critical value, we also obtain the action of the fundamental group of  $\mathbb{C} \setminus V(h)$  on  $h^{-1}(b_0)$ .

In the case that the isotropy subgroup of the action is a normal subgroup, we have that the Galois group of the polynomial is the quotient of the fundamental group of  $\mathbb{C} \setminus V(h)$  induced by the isotropy normal subgroup. This gives a relation with the Galois theory of field extensions that could permit to verify whether the numerical roots of the polynomial  $h(x) - b_0 = 0$  can be solved in a symbolic way.

Therefore, this algorithm based on Fox's theory of branched coverings and Newton's numerical methods could be used to establish nice connections between the finite set of numerical solutions of the equation  $h(x) = b_0$  enriched with the action of the fundamental group and the possible symbolic solutions obtained by means of radicals.

University of La Rioja

*E-mail address:* `luis-javier.hernandez@unirioja.es`

*E-mail address:* `miguel.maranon@unirioja.es`

*E-mail address:* `maria-teresa.rivas@unirioja.es`